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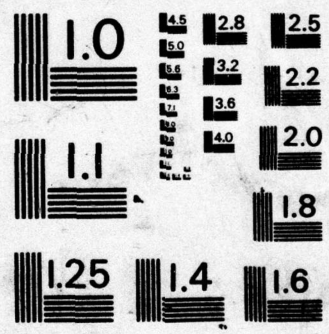

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FREQUENCY DOMAIN INTERPOLATION

By

C. T. Leondes, Fellow, IEEE

And

D. D. Rivers

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ABSTRACT

A formula is derived for interpolation between output samples of an FFT, i.e., in the frequency domain. Such a formula is useful for obtaining greater frequency resolution when two coarse FFT outputs are available. Consideration is also given to the effect of such interpolation on a weighted FFT.

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In digital signal processing applications involving spectral measurements, frequency resolution is limited by the length of the input sample stream and by available data processing capacity. However, in applications such as doppler radar, it often happens that high frequency resolution is needed only over regions which are small compared to the sampling rate, whereas coarser information will do for the rest of the spectrum. Such increased resolution may be achieved in a number of different ways:

(a) One may increase the order of the filter bank Fast Fourier Transform (FFT). Normally the order is chosen to be a radix 2 number in order to optimize the efficiency of the FFT. Thus, if the order is doubled, the processing time more than doubles before any information becomes available.

(b) If the frequency regions of interest are preselectable, high resolution may be obtained if they are small (see, for example, p. 390 of [1]). The breakeven point turns out to be a region size of half the sampling rate. This method is not universally applicable in that no coarse information is made available for the full range of frequencies, the frequency regions must be selected apriori (rather than as a result of processing coarse data), and the region sizes must be simple fractions of the sampling rate such as 1/3, 1/12, etc.

(c) Another alternative is to form two low order (say N) FFTs from the input data, and then combining (over the frequency regions of interest only) to obtain the higher resolution 2Nth order FFT. The first FFT would use the even numbered samples, and the second the odd, as in the mechanization of a decimation-in-time FFT. The drawback with this method is that the outputs of the primary FFTs will furnish coarse spectral data aliased 2:1. Thus, with the first half of the spectrum folded into the second half, the primary FFTs will yield intelligible coarse information only if the spectral regions of interest are folded into regions which are clear (i.e. which contain noise only). Even when this happens to be the case, the aliasing reduces the coarse data signal-to-noise ratio by 3 db. Again, no coarse spectral data would be available before 2N data points were collected and processed.

(d) Finally, one could perform an Nth order FFT on the first N samples and another on the next N. The outputs of either of these can, of course, be used as coarse data to determine regions where greater resolution is required. (In doppler radar applications, for example, targets could be detected with the coarse data). The question now arises: can the two Nth order FFT outputs be easily combined to produce greater resolution in frequency areas of interest? The answer is "yes". We now derive the formula to be used in this data combination.

Let the two sequential FFT output vectors be FFT1 and FFT2 respectively. Let the 2N inputs be

$$\left\{ x_n \ , n = 0, \cdots, 2N-1 \right\}, \tag{1}$$

and define

$$W_M \triangleq e^{-j\frac{2\pi}{M}} \tag{2}$$

Let the high resolution spectrum be

$$\left\{ a_k \ , k = 0, \cdots, 2N-1 \right\}. \tag{3}$$

Then, thinking in terms of a decimation-in-frequency FFT, we see that the even outputs are

$$\left\{ a_{2k} = \sum_{n=0}^{N-1} y_n \, W_N^{nk} \ , \ k = 0, \cdots, N-1 \right\} \tag{4}$$

where

$$\left\{ y_n \triangleq x_n + x_{n+N} \ , n = 0, \cdots, N-1 \right\}. \tag{5}$$

Thus

$$\left\{ a_{2k} = FFT1_k + FFT2_k \ , k = 0, \cdots, N-1 \right\} \tag{6}$$

are the even members of the output.

The odd members are

$$\left\{a_{2k+1}=\sum_{n=0}^{N-1}\left[z_n W_{2N}^{n}\right] W_N^{nk}, \quad k=0,\cdots,N-1\right\} \tag{7}$$

where

$$\left\{z_n \triangleq x_n - x_{n+N} \quad , \quad n=0,\cdots,N-1\right\}. \tag{8}$$

Thus

$$a_{2k+1}=\sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{l=0}^{N-1}\left(FFT1_l - FFT2_l\right)W_N^{-ln}\cdot \cdot W_{2N}^{n}\right]W_N^{nk} \quad , k=0,\cdots,N-1 \tag{9}$$

Now define

$$\left\{d_k \triangleq FFT1_k - FFT2_k \quad , \quad k=0,\cdots,N-1\right\} \tag{10}$$

and observe that

$$W_{2N}^{n} = W_N^{\frac{n}{2}}. \tag{11}$$

We obtain

$$a_{2k+1}=\sum_{n=0}^{N-1}\left\{\left[\frac{1}{N}\sum_{l=0}^{N-1} d_l W_N^{-nl}\right]W_N^{\frac{n}{2}}\right\} W_N^{nk}$$

$$=\frac{1}{N}\sum_{l=0}^{N-1} d_l \sum_{n=0}^{N-1} W_N^{\left(\frac{1}{2}-l+k\right)n}$$

$$=\frac{1}{N}\sum_{l=0}^{N-1} d_l \left[\frac{1-W_N^{\left(\frac{1}{2}-l+k\right)N}}{1-W_N^{\frac{1}{2}-l+k}}\right] \quad ,$$

and thus

$$\left\{ a_{2k+1} = \frac{1}{N} \sum_{l=0}^{N-1} \frac{2\, d_l}{1 - W_N^{\frac{1}{2} - l + k}} \quad , \; k = 0, \cdots, N-1 \right\} . \tag{12}$$

The frequency domain interpolation formula we require consists of eqs. (6) and (12). The formula is exact, but in order to gain the computational advantage which is desired in interpolation formulae, one would want to sum over only a limited range, say, for $l \in [L_1, L_2]$, where L_1 and L_2 are close to k. Note that the importance of the $\{d_l\}$ to the spectral estimate a_{2k+1} as approximately inversely proportional to $l-k$. In fact, if N is large, we see that

$$\begin{aligned} 1 - W_N^{\frac{1}{2} - l + k} &= 1 - e^{-j 2\pi \left[\frac{\frac{1}{2} - l + k}{N} \right]} \\ &\approx j \frac{\pi}{N} (1 - 2l + 2k) \end{aligned} \tag{13}$$

whenever we use only a few neighboring d_l such that

$$\frac{\pi}{N} (1 - 2l + 2k) \ll 1 .$$

If this is done, we may set

$$a_{2k+1} \approx \frac{2}{j\pi} \sum_{l=L_1}^{L_2} \frac{d_l}{1 - 2(l-k)} . \tag{14}$$

Eq. (14) may be used as an approximation to the odd numbered values, eq. (12). However, for a given application there is not a particularly great advantage in doing so. The coefficients of d_l given in eq.(12) are easily obtained on a computer.

At this point one might ask how big the interval $[L_1, L_2]$ should be, and if the terms outside this interval may indeed reasonably be neglected. Since the sum in eq.(12), or equivalently, in the approximating eq.(14), is finite, there is no question of divergence. However, the basic question, "How good is the approximation?", is probably best answered by deriving a formula for the finite impulse response (FIR) filter weights corresponding to the interpolated values, and then checking their frequence response.

Using nearest neighbors only, eq. (12) becomes

$$a_{2k+1} = a_1 d_k + \alpha_1^* d_{k+1} \tag{15}$$

where

$$\alpha_1 \triangleq \frac{2}{N(1 - W_N^{\frac{1}{2}})} \tag{16}$$

and * denotes the complex conjugate.

Thus

$$\begin{aligned}
a_{2k+1} &\approx \alpha_1 (FFT1_k - FFT2_k) \\
&\quad + \alpha_1^* (FFT1_{k+1} - FFT2_{k+1}) \\
&= \sum_{n=0}^{N-1} x_n \left[\alpha_1 W_N^{nk} + \alpha_1^* W_N^{n(k+1)}\right] \\
&\quad - \sum_{n=0}^{N-1} x_{n+N} \left[\alpha_1 W_N^{nk} + \alpha_1^* W_N^{n(k+1)}\right] \\
&= \sum_{n=0}^{N-1} x_n \left[\alpha_1 W_N^{nk} + \alpha_1^* W_N^{n(k+1)}\right] \\
&\quad - \sum_{n=N}^{2N-1} x_n \left[\alpha_1 W_N^{(n-N)k} + \alpha_1^* W_N^{(n-N)(k+1)}\right]
\end{aligned}$$

$$= \sum_{n=0}^{2N-1} x_n \, \mathrm{sgn}\,(N-n-1) \left[\alpha_1 W_N^{nk} + \alpha_1^* W_N^{n(k+1)}\right] \tag{17}$$

where

$$\mathrm{sgn}(m) \triangleq \begin{cases} -1 & m < 0 \\ +1 & m \geq 0 \end{cases} \tag{18}$$

Viewing eq. (17) as a convolution sum, we see that the FIR weights are

$$\left\{\beta_n^1 = \mathrm{sgn}(n-N)\left[\alpha_1 W_N^{-(n+1)k} + \alpha_1^* W_N^{-(n+1)(k+1)}\right], n=0,\cdots,2N-1\right\} \tag{19}$$

Generalizing the definition (16) to

$$\alpha_m \triangleq \frac{2}{N\left(1-W_N^{-\frac{1}{2}+m}\right)} \quad , \tag{20}$$

we can write an expression for the FIR weights using the nearest M neighbors. We first set

$$a_{2k+1} \approx \sum_{m=0}^{M-1}\left[\alpha_{m+1} d_{k-m} + \alpha_{m+1}^* d_{k+1-m}\right]. \tag{21}$$

Comparing eqs. (20) with eqs. (15) and following, we easily deduce that the weights are

$$\begin{aligned} \Big\{ \beta_n^M = \mathrm{sgn}(n-N)\sum_{m=0}^{M-1}\Big[\alpha_{m+1} W_N^{-(n+1)(k-m)} \\ + \alpha_{m+1}^* W_N^{-(n+1)(k+1+m)}\Big], n=0,\cdots,2N-1\Big\}. \end{aligned} \tag{22}$$

If the approximation leading to eq. (14) is used, we obtain

$$\begin{aligned} \Big\{\beta_n^M \approx \frac{1}{j\pi} \mathrm{sgn}(n-N)\sum_{m=0}^{M-1}\frac{1}{1+2m}\Big[W_N^{-(n+1)(k-m)} \\ - W_N^{-(n+1)(k+1+m)}\Big], \quad n=0,\cdots,2N-1\Big\} \end{aligned} \tag{23}$$

for the FIR weights using the M nearest neighbors. It is desirable to normalize the weights as derived in eqs. (19), (22), or (23) in order to achieve a specific gain at the center of the filter. In the following example we show magnitude responses of a few candidate interpolating filters. They are all normalized to OdB.

Example

Suppose N = 16, 2N = 32, and k = 2. I.e., we desire the spectral estimate $a_{2k+1} = a_5$ based on two sequential 16-point FFT output vectors. (Such a small order is chosen here for illustrative purposes only.) Figure 1 shows how the high resolution spectrum is formed using nearest neighbors only.

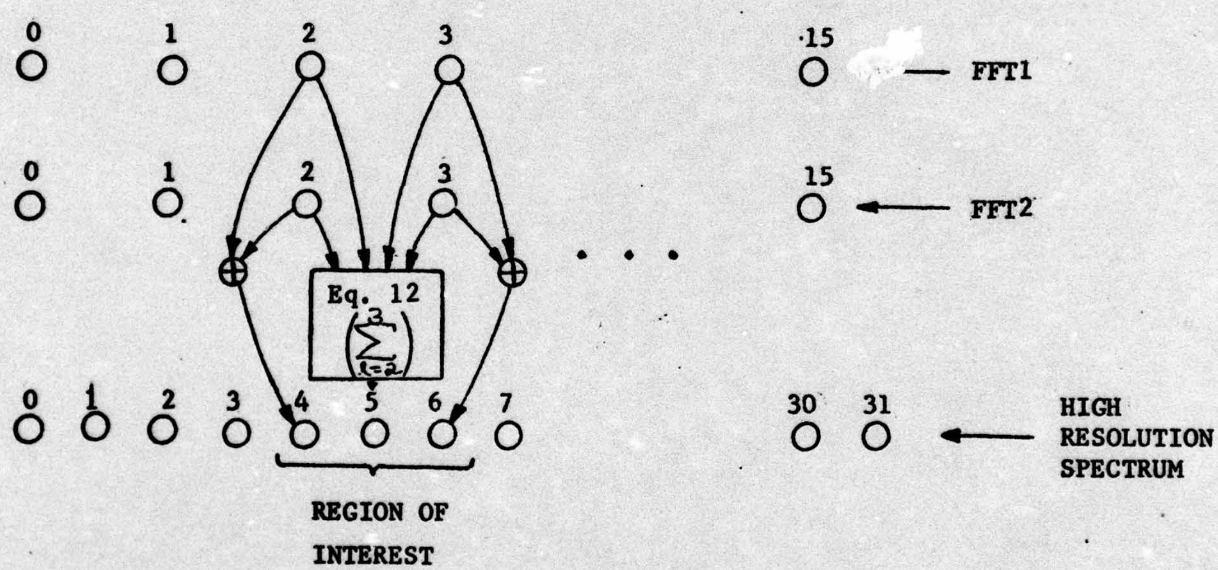


Figure 1. Interpolation with nearest Neighbors only.

Figure 2 shows the result of using nearest neighbors only, formula (19). In this case N Dolph-Chebyshev weights with 40 dB sidelobe suppression were used [2]. Call these weights $\{w_n, n=0, \cdots, N-1\}$.
The equivalent set of 2N weights becomes

$$\{w'_n, n=0, \cdots, 2N-1\} \triangleq \{w_0, \cdots, w_{N-1}, w_0, \cdots, w_{N-1}\}.$$

The filters corresponding to the neighboring even-numbered spectral estimates, a_4 and a_6, are represented by dashed curves in the figure. As expected, using N weights leads to rather poor filters after combining FFT1 and FFT2. Note that such preweighting, which is commonly used for better filter shaping, is easily incorporated into the formulas derived above. Since the weights go with the input samples, equations (6) and (12) remain unchanged, and (22) becomes

$$\Big\{\beta_n^{M'} = w'_{2N-1-n}\,\mathrm{sgn}\,(n-N)\sum_{m=0}^{M-1}\Big[\alpha_{m+1} W_N^{-(n+1)(k-m)} + \alpha_{m+1}^{*} W_N^{-(n+1)(k+1+m)}\Big],\quad n=0,\cdots,2N-1\Big\}. \tag{24}$$

The magnitude response of these $\{\beta_n^{M'}\}$ is what is graphed in figure 2. This kind of weighting is appropriate for the two N-point FFTs, but is undesirable for a 2N-point FFT. The appropriate weighting would consist of 2N weights. Figure 3 shows a_4, a_5 and a_6 in this case. The interpolating filter's sidelobe structure is now much more appealing, and, of course, the even numbered filters are of just the desired Dolph-Chebyshev type. Figure 4 shows the same situation, but in this case the four nearest neighbors on each side have been used (formula (24) with $\{w_n'\}$ as 2N Dolph-Chebyshev weights.) As expected the result is much improved. The drawback with using 2N weights is obvious: each of the primary FFTs receives a lopsided set of weights. However, since these coarse spectra are used for quick-look purposes only, their degraded quality may often be tolerable. Figure 5 shows the magnitude response of $FFT1_0$, the first filter. The dashed curve shows the filter shape with N Dolph-Chebyshev weights.

Conclusion

This paper has derived a set of equations, (6) and (12 or 14), useful for interpolating between weighted (or unweighted) output samples of sequential FFTs, i.e., in the frequency domain. The quality of the interpolation has been illustrated by means of a numerical example, which also showed the consequences of two different weighting approaches. It is challenging to consider that a compromise may exist between these two approaches, i.e., a weighting sequence which would improve both the high resolution and the quick-look spectra as compared to the results of either of the above approaches.

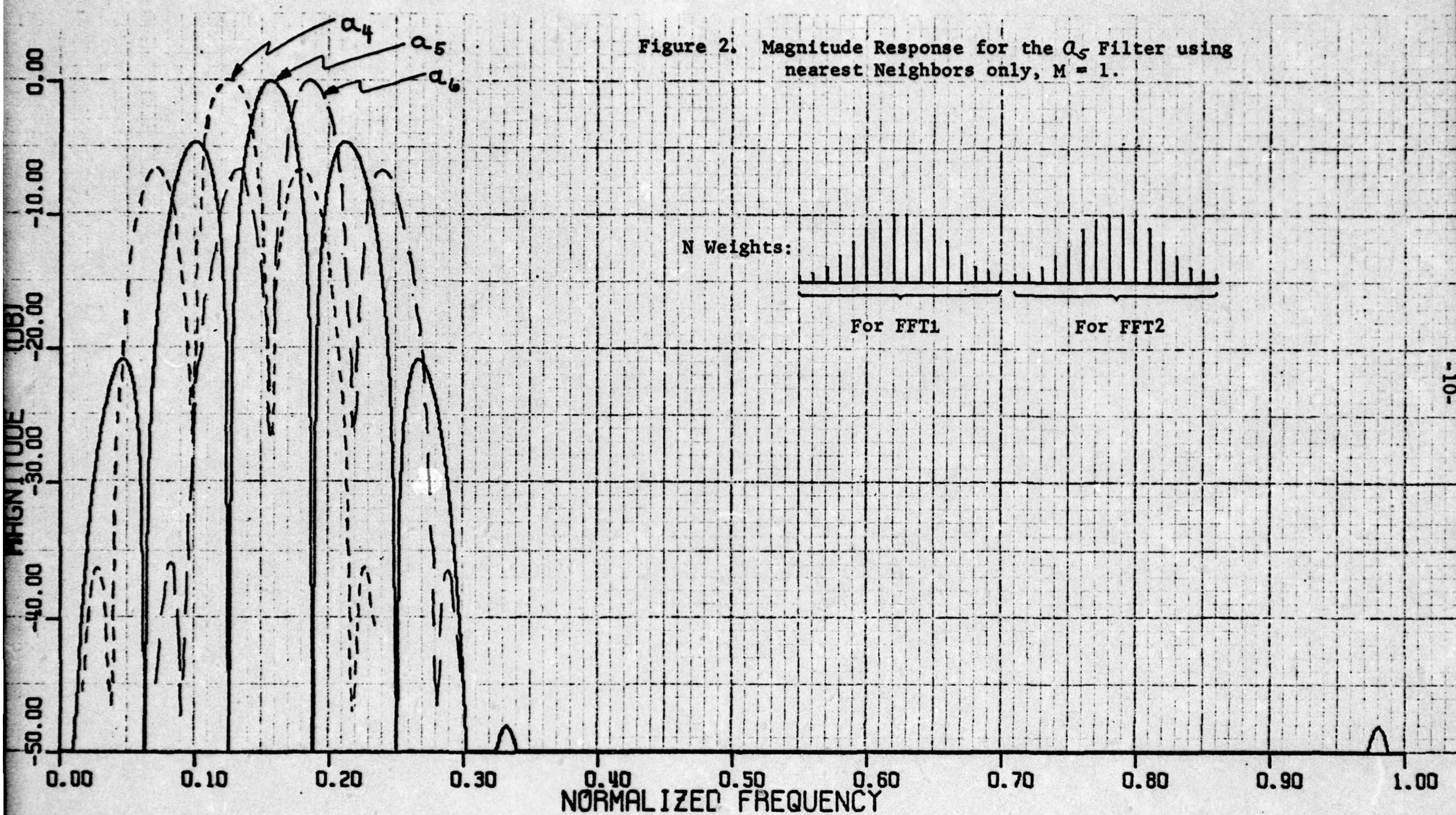


Figure 2. Magnitude Response for the a_5 Filter using nearest Neighbors only, M = 1.

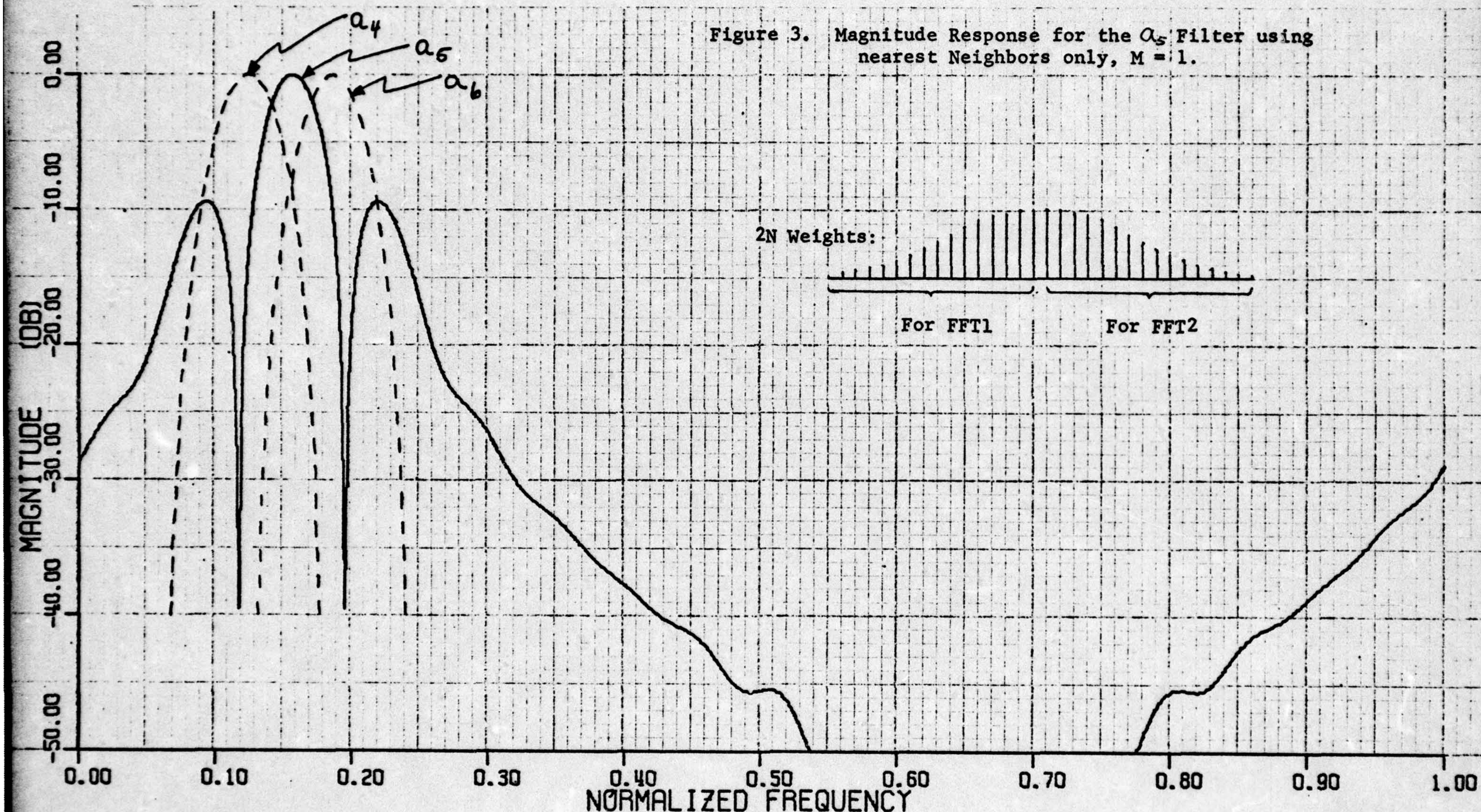


Figure 3. Magnitude Response for the a_5 Filter using nearest Neighbors only, M = 1.

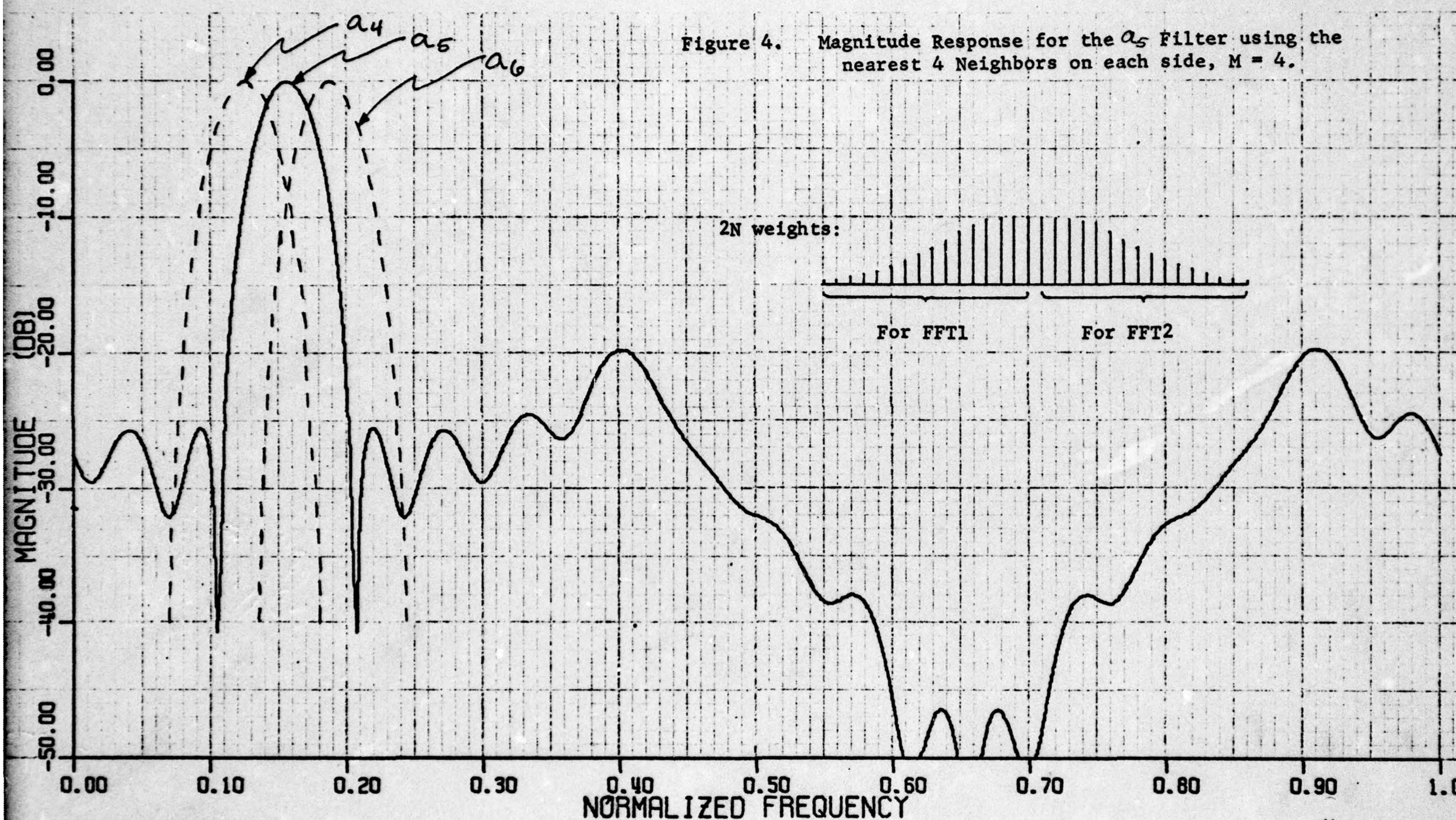
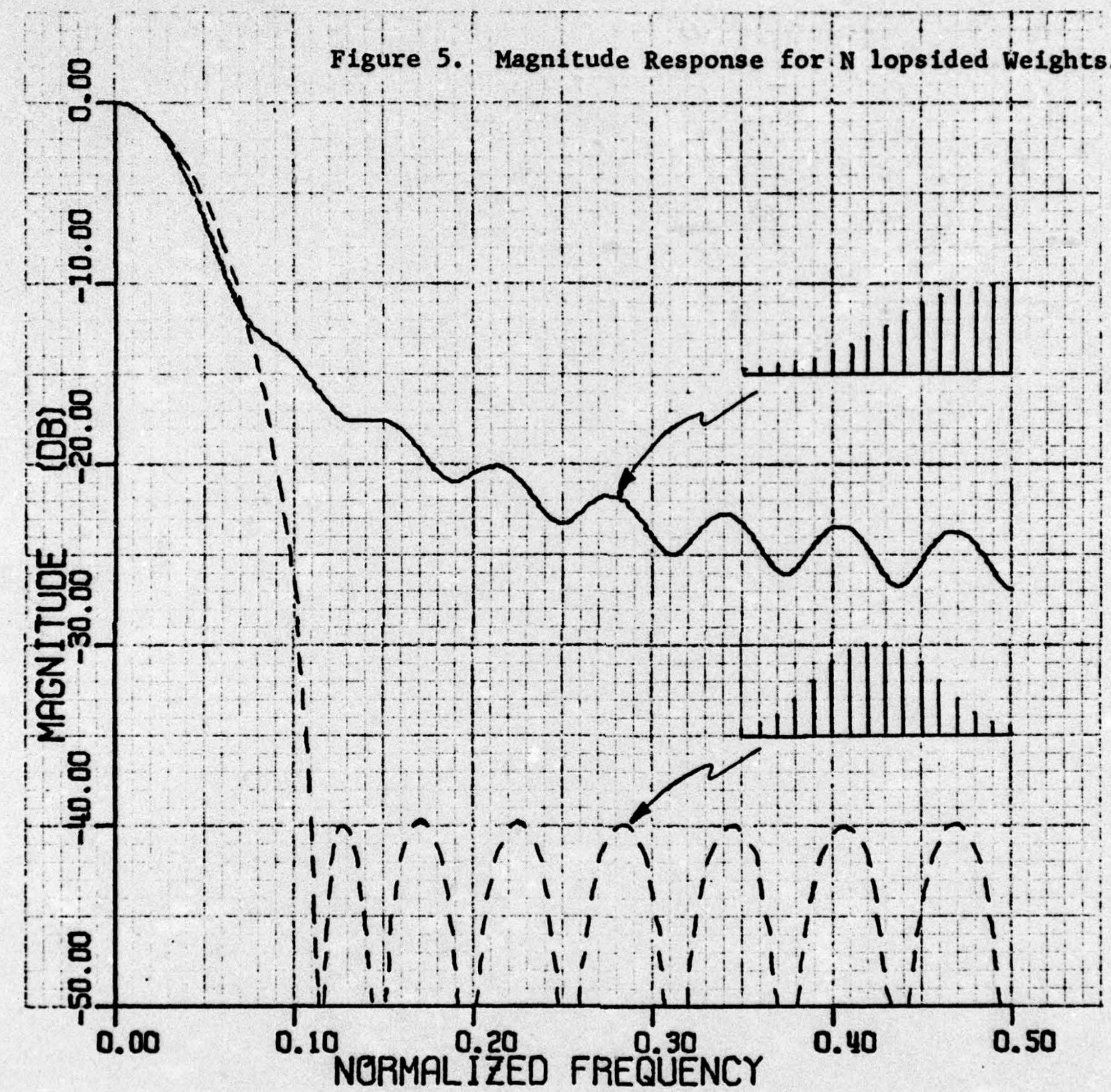


Figure 4. Magnitude Response for the a_5 Filter using the nearest 4 Neighbors on each side, M = 4.

Figure 5. Magnitude Response for N lopsided Weights.



REFERENCES

[1] L. R. Rabiner and B. Gold, "Theory and Application of Digital Signal Processing," Prentice-Hall, Inc., Englewood Cliffs, New Jersey 1975.

[2] H. R. Ward, "Properties of Dolph-Chebyshev Weighting Functions," IEEE Trans. of Aerospace and Elec. Sys., September 1973, pp. 785-786.

Cornelius T. Leondes (S'48-M'52-SM'57-F'69) was born on July 21, 1927. He received the B.S.E.E., M.S.E.E., and Ph.D. degrees from the University of Pennsylvania, Philadelphia, in 1949, 1951, and 1954, respectively.

He is now a Professor with the Department of Engineering Systems, School of Engineering and Applied Science, University of California, Los Angeles, and serves as consultant or member of numerous national technical and scientific advisory boards. He is editor and co-author of several textbooks on systems techniques.

Dr. Leondes is a member of the NATO-AGARD Guidance and Control Technical Panel. During 1962-1963 he was a Guggenheim Fellow and Fulbright Research Scholar. In 1970 he was the recipient of the IEEE Baker Prize Award. In 1973 he was the recipient of the IEEE G-AES Barry Carlton Honorable Mention Award.

Daniel D. Rivers was born in Sauda, Norway, on July 20, 1945. He received the B.A. degree in mathematics and physics from Luther College, Decorah, Iowa, 1967, and the M.A. degree in mathematics from the University of California at Los Angeles 1968. He is currently a candidate for the Ph.D. in mathematics at UCLA studying under a Hughes Aircraft Company Doctoral Fellowship in the field of digital signal processing. Mr. Rivers is employed at Hughes Aircraft Company, Culver City, California. He is presently with the DATA PROCESSING SYSTEMS ~~Computer Applications~~ Department, Strategic Systems Division. Mr. Rivers is a member of Pi Mu Epsilon mathematical honor society.

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